[This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6216**

Unique Paper Code : **222502** **D**

Name of the Paper : **Quantum Mechanics (PHHT-516)**

Name of the Course : **B.Sc. Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all. Question No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

Symbols have their usual meaning.

1. Attempt any *five* of the following : $3 \times 5 = 15$

(*a*) A metal surface when irradiated with light of wavelength 5896 Å emits electrons for which the stopping potential is 0.12 V. When the same surface is irradiated with 2830 Å, it emits electrons for which the stopping potential is 2.20 V. Calculate the value of Planck's constant.

P.T.O.

(*b*) Compare the de Broglie wavelengths for an electron with a kinetic energy of 1 eV and a ball of mass 300 gm travelling at 100 km/hr.

(*c*) Determine the smallest possible uncertainty in the position of an electron moving with velocity 3×10^7 m/s.

(*d*) Establish time independent form of Schrödinger equation for stationary states.

(*e*) Determine the probability of finding a particle of mass m between $x = 0$ and $x = L/10$, if the particle is described by the normalized wave function :

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

for $0 \leq x \leq L$ and is in the $n = 3$ state.

(*f*) A radial function in spherical polar coordinates is :

$$R_n(r) = Ce^{-r/2}\, U_n(r),$$

where C is a normalization constant. Discuss the physical acceptability of $R_n(r)$ if $U_n(r)$ behaves as :

(*i*) $1/r^2$ for small values of r and as a polynomial in r otherwise; and

(*ii*) a polynomial in r of degree more than 3 for all values of r.

(g) A one-dimensional harmonic oscillator is in a state described by the wave function :

$$\psi(x, 0) = \frac{1}{2}\psi_0(x) + \frac{i}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}e^{i\pi/3}\psi_2(x)$$

where $\psi_n(x)$ are the usual normalized orthogonal wave functions. Normalize the wave function $\psi(x, 0)$.

(h) Find the classical amplitude of a one-dimensional harmonic oscillator in its ground state with an energy $\frac{1}{2}\hbar\omega$.

2. (a) In the Compton scattering of a photon of frequency ν by a free electron through an angle □, using the expressions for momentum conservation : 10

$$pc\cos\theta = h\nu - h\nu'\cos\phi \text{ and } pc\sin\theta = h\nu'\sin\phi$$

and the expression for change in wavelength of scattered photon :

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\phi)$$

prove that :

$$\tan\theta = \frac{1}{[1+\beta]\tan\frac{\phi}{2}}; \text{ where } \beta = \frac{h\nu}{m_o c^2}$$

(*b*) A metal surface emits electrons with maximum kinetic energies E_1 and E_2 when illuminated with light of wavelengths λ_1 and λ_2 respectively, where $\lambda_1 > \lambda_2$. Prove that the Planck's constant h and work function ϕ of the metal are given by :

$$h = \frac{(E_2 - E_1)\,\lambda_1\,\lambda_2}{c(\lambda_1 - \lambda_2)} \text{ and } \phi = \frac{E_2\,\lambda_2 - E_1\,\lambda_1}{(\lambda_1 - \lambda_2)}.$$

3. (*a*) Explain de Broglie hypothesis for matter waves. Show that for the de Broglie wave associated with a moving particle the group velocity is equal to the particle velocity. 5

(*b*) Assume that at time $t = 0$, a single non-interacting electron is located near $x = x_0$ with the probability P dx of finding it between x and $x + dx$ being given by :

$$\psi(x, 0) = A e^{-(x - x_0)^2/2a^2}\, e^{ip_0 x/\hbar}$$

Obtain the expectation values of x and p. Also show that : 10

$$\Delta x \,.\, \Delta p = \hbar/2.$$

4. (*a*) Describe an experiment to locate the position of a free electron by a microscope using γ ray and hence, obtain an expression for uncertainty principle. 10

(*b*) Determine the minimum uncertainty in the position of a particle in terms of de Broglie wavelength when the uncertainty in the velocity of a particle in one-tenth of its velocity. 5

5. (*a*) Obtain the energy eigen values and the normalized wave functions for a free particle of mass m trapped in a one-dimensional box of length L along x-axis in positive direction of x from the origin. 10

(*b*) The wave function of a particle confined in a box of linear dimension L along x-axis is :

$$\psi(x) = Ae^{i\alpha x}; \; 0 \leq x \leq L.$$

Find the probability of finding the particle in the distance $0 \leq x \leq \frac{L}{4}$. 5

6. Solve the time independent Schrödinger equation for the energy levels of a one-dimensional harmonic oscillator. Draw the energy level diagram. Explain the physical significance of zero-point energy. 15

7. A particle of mass m and energy E moves along x-axis from a region of zero potential towards a one-dimensional step potential barrier of height V_0 of infinite extent. Assuming $E > V_0$, derive expressions for the reflection and transmission coefficients. Comment on the wavelengths associated with the incident, reflected and transmitted waves. Also, obtain expressions for probability current densities associated with the incident, reflected and transmitted waves. 15

8. (a) Solve the angular equation of hydrogen atom in spherical polar coordinates, given as :

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 y}{\partial\phi^2}+\lambda y=0$$

to obtain relation $\lambda = l(l+1)$. 11

(b) Given the wave function of ground state of hydrogen atom is :

$$\psi_{100}(r)=\frac{e^{-r/a_0}}{(\pi a_0^3)^{1/2}},$$

where the symbols have usual meaning and :

$$a_0=\frac{4\pi\in_0\hbar^2}{me^2}.$$

Calculate the most probable distance of electron from nucleus in ground state. 4

$\hbar = 1.054 \times 10^{-34}$ Js

$h = 6.63 \times 10^{-34}$ Js

Rest mass of electron = 9.1×10^{-31} kg

Charge of electron = 1.6×10^{-19} C

Rest mass energy of electron = 512 KeV

Velocity of light in free space = 3×10^8 m/s.

Dec 2012

This question paper contains 3 printed pages. Your Roll No. ....

Sl. No. Of Ques. Paper : 8421C
Unique Paper Code : 222502
Name of the Paper : PHHT-516 : Quantum Mechanics
Name of the Course : B.Sc. (Hons.) Physics Part III
Semester : V
Duration : 3 hours
Maximum Marks : 75

Attempt five questions in all. Question No. 1 is compulsory.
Use of non-programmable scientific calculators is allowed.
Symbols have their usual meaning.

Q. 1 *Attempt any five of the following.*

a) Starting from Heisenberg's Uncertainty Principle $\Delta x \Delta p \geq \hbar/2$, obtain a similar inequality for the variables x and λ, where λ is the de Broglie wavelength.

b) The wave function for a particle confined in a one-dimensional box of length L is given by

$$\psi(x) = A\sin(n\pi x / L).$$

Normalize the wave function and evaluate the expectation values of its momentum.

c) Determine which of the following wave functions are physically acceptable solutions of the Schrodinger wave equation:

(1) tan x (2) sin x (3) $1/x$ (4) $\exp(-x^2/2)$ (5) sec x (6) $\exp(ikx)$

d) An X-ray photon undergoes Compton scattering by 90^0. If the frequencies of the incident and the scattered photons are υ and υ' respectively, calculate the de Broglie wavelength associated with the recoil electron.

e) The uncertainty in the velocity of a particle is equal to its velocity. Calculate the minimum uncertainty in the position of the particle in terms of its de Broglie wavelength.

f) Calculate the group velocity of ocean waves whose phase velocity is given by $v_p = \sqrt{\frac{g\lambda}{2\pi}}$, where λ is wavelength of ocean waves and g the acceleration due to gravity.

g) Explain what is meant by space-quantization of angular momentum L. What role does the magnetic quantum number m_l play in this quantization?

h) Let an electron be regarded as a linear harmonic oscillator with angular frequency 5×10^{14}/sec . Calculate: (i) its zero-point energy and (ii) the classical limits of its motion in the n = 1 state.

(3 x 5)

Q. 2 a) Why is the classical wave theory unable to explain the observations of the photoelectric effect? How does Einstein's photoelectric equation resolve these difficulties?

b) Show that a free electron cannot completely absorb a photon and conserve both energy and momentum.

c) The stopping potential for photoelectrons emitted from a surface illuminated by light of wave length $\lambda = 4900\ A^0$ is 0.7 V. When the incident light is changed, the stopping potential changes to 1.41 V. What is the new wavelength?

(8, 3, 4)

Q. 3 a) Explain why it is necessary to create a wave packet to describe a particle in quantum mechanics. Construct a wave packet using two simple harmonic waves and obtain Heisenberg's uncertainty principle connecting position and momentum of a particle.

b) Show that the Uncertainty principle can be used to derive an expression for the radius of the first Bohr orbit of the Hydrogen atom.

(2, 7, 6)

Q. 4 a) Explain the de Broglie hypothesis for matter waves. Describe the Davisson Germer experiment in detail and point out how it established the wave nature of electrons.

b) Calculate the de Broglie wave length for an electron with kinetic energy of

i) 1 eV and ii) 1 MeV.

(Rest energy of an electron = 0.511 Mev)

(10, 5)

Q. 5 a) Write down the time dependent one-dimensional Schrödinger equation for a free particle of mass m. Show that the states with definite energy E can be represented by the wave function $\Psi(x,t) = \Phi(x) \exp(-iEt/\hbar)$, where $\Phi(x)$ is the solution of the time independent Schrödinger equation.

b) Obtain the energy eigenvaiues and the normalized wave functions for a free particle of mass m trapped in a one dimensional box of length L

(8, 7)

Q. 6 a) A beam of particles, each of mass m and energy E, moving along the x-axis is incident on a regular potential barrier of width a and height V_0 given by

$$V(x) = 0 \qquad x < 0$$
$$= V_0 \qquad 0 < x < a$$
$$= 0 \qquad x > a$$

Obtain an expression for the transmission coefficient (T) for the case $E < V_0$. Hence derive the limiting expression for T for a very broad, high barrier.

b) What do you understand by tunnel effect? Explain with reference to the Tunnel diode.

(12, 3)

Q. 7 a) Set up the time-dependent Schrodinger equation for a linear harmonic oscillator and obtain an expression for the energy eigenvalues of the oscillator. Draw the energy level diagram.

b) Comment on the statement that: "Zero point energy of a harmonic oscillator is in agreement with the Uncertainty Principle."

(12, 3)

Q. 8 a) Using the radial equation for hydrogen atom

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{2m}{\hbar^2}\left\{E+\frac{e^2}{4\pi\varepsilon_0 r}-\frac{l(l+1)\hbar^2}{2mr^2}\right\}R=0,$$

obtain an expression for the energy eigenvalues.

b) The ground state wave function for hydrogen is

$$\psi(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}.$$

Calculate the probability of finding the electron at a distance less than a_0

(10, 5)

Physical Constants:

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

[This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 926 27 NOV 2014

Unique Paper Code : 222502 E

Name of the Paper : **Quantum Mechanics (PHHT-516)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours**

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all. Question No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

All symbols have their usual meanings.

1. Attempt any *five* of the following : $3\times5=15$

(*a*) The threshold frequency of photoelectric emission for a certain metal is 10^{16} Hz. Calculate the cut-off wavelength and the work function.

(*b*) Calculate the maximum percentage change in the wavelength of a 2000 Å photon scattered by an electron.

(*c*) A microscope using photons is employed to locate an electron in an atom to within a distance of 0.2 Å. Determine the uncertainty in the velocity of the electron located in this manner.

P.T.O.

(*d*) Describe the conditions for physical acceptability of a wave function.

(*e*) For the normalized wave function $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n \pi x}{L}$ for $0 \le x \le L$, compute the expectation values of position $\langle x \rangle$ and momentum $\langle p \rangle$ where the symbols have the usual meanings.

(*f*) Determine the potential function V(x) for a particle of mass m and energy E if the wave function $\psi(x) = Ae^{-ikx}$ (where A and k are constants) associated with the particle is a solution of the time-independent one-dimensional Schrödinger equation.

(*g*) The energy of a one-dimensional linear harmonic oscillator in its third excited state is 0.1 eV. Find the frequency of vibration.

(*h*) A radial function in spherical polar coordinates is $R_n(r) = Ce^{-r/2} U_n(r)$, where C is a normalization constant. Discuss the physical acceptability of $R_n(r)$ if $U_n(r)$ behaves as :

(*i*) Constant for small values of r and as e^r for large values of r and

(*ii*) Constant for small values of r and as $e^{r/3}$ for large values of r.

2. (*a*) Describe an experiment to demonstrate the photoelectric effect. Explain the characteristics of the effect. How did Einstein explain the phenomenon ? 10

(*b*) The photoelectric threshold wavelength of silver is 2762 Å. Calculate the maximum kinetic energy of photoelectrons and the stopping potential in volts for the electrons when the silver surface is illuminated with light of wavelength 2000 Å. 5

3. (*a*) Explain Compton effect. Obtain an expression for Compton wavelength. Di expression. 10

(*b*) A photon of wavelength 3 Å suffers Compton scattering by a free electron originally at rest. Determine the kinetic energy of recoil electron if the angle of scattering of photon is 90°. 5

4. (*a*) Explain de-Broglie hypothesis of matter waves. Explain the concept of wave function and probability density of a particle. Obtain an expression for the velocity of a wave packet associated with a moving particle. 10

(*b*) Show that the de Broglie wavelength λ associated with an electron of rest mass m_0 and kinetic energy K under relativistic condition is $\lambda = \dfrac{hc}{\sqrt{K(K + 2m_0 c^2)}}$ Å. 5

5. (*a*) Given a wave function $\psi(x) = A\left(\dfrac{\pi}{\alpha}\right)^{-1/4} e^{-\alpha x^2}$, for $-\infty < x < \infty$. Determine the value of A by normalizing the wave function and hence, calculate $\Delta x = \sqrt{<x^2> - <x>^2}$. 10

(*b*) A typical atomic nucleus is about 5×10^{-15} m in radius. Use the uncertainty principle to set a lower limit on the energy of an electron if it is to be part of a nucleus. 5

6. Solve the time independent Schrödinger equation for the energy levels of a one-dimensional harmonic oscillator. Draw the energy level diagram. Explain the physical significance of zero-point energy. 15

P.T.O.

7. (*a*) A particle of mass m and energy E moves along x-axis from a region of zero potential towards a one-dimensional step potential barrier of height V_0 of infinite extent. Assuming $E < V_0$, derive expressions for the reflection and transmission coefficients. 10

(*b*) Explain the working of a tunnel diode. 5

8. Solve the angular equation of hydrogen atom in spherical polar coordinates, given as $\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}+\lambda Y=0$ to obtain permissible values of m_l and the relation $\lambda = l(l+1)$. Using this solution, obtain the expressions for eigen values of L^2 and L_Z where $L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$ and $L_Z = -i\hbar\frac{\partial}{\partial\phi}$ are in operator form in spherical polar coordinates. 15

$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

Rest mass of electron $= 9.1 \times 10^{-31}$ kg

Charge of electron $= 1.6 \times 10^{-19}$ C

Rest mass energy of electron $= 512$ KeV

Velocity of light in free space $= 3 \times 10^8$ m/s

[estion paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

26 NOV 2014

of Question Paper : **1257**

e Paper Code : **222563**

E

of the Paper : **Physics-V [Quantum Mechanics & Atomic Physics] (PHPT-505)**

of the Course : **B.Sc. (Physical Science)**

ster : **V**

ion : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions.

All questions carry equal marks.

(*a*) Describe Compton scattering and obtain an expression for the change in wavelength of scattered photon. 2,9

(*b*) Define Compton wavelength and find out its value. 2,2

(*a*) Explain de-Broglie hypothesis. Derive an expression for de-Broglie wavelength. Give an experimental verification for the existence of matter waves on the basis of Davisson and Germer experiment. 4,6

(*b*) Discuss Gamma-ray microscope experiment. 5

P.T.O.

3. (*a*) Give expressions for any *three* basic operators in quantum mechanics.

(*b*) What do you understand by normalization of a wave function ?

(*c*) Derive Schrodinger's time independent form of wave equation for a particle in o

dimension.

4. (*a*) Solve Schrodinger equation for a particle in a one-dimensional box. Obtain the express

for normalized wave functions and energy eigenvalues.

(*b*) A particle is moving in a one-dimensional box of width 10 Å. Calculate the probabilit

of finding the particle within an interval of 1 Å at the centre of the box, when it is i

its state of least energy.

5. (*a*) Explain space quantization of $\vec{L}$ and $\vec{S}$ with the help of an example.

(*b*) Name the different series present in the spectra of alkali atoms.

(*c*) What do you understand by symmetric and anti-symmetric wave functions ?

6. (*a*) Explain with the help of Stern-Gerlach Experiment the existence of spin in a

atom.

(*b*) What is a Bohr Magneton ? Give its unit.

12

7. (*a*) What is Normal Zeeman Effect ? Derive an expression for the frequency shift in Normal Zeeman Effect. 3,9

(*b*) A sample of a certain element is placed in a 0.300 T magnetic field and suitably excited. How far apart are the Zeeman components of the 450 nm spectral line of this element ? 3

8. (*a*) Explain LS coupling. 4

(*b*) What do you understand by spin-orbit coupling ? What is its significance ? 4,3

(*c*) Obtain an expression for the maximum number of electrons that can be accommodated in a shell or an orbit. 4

Some constants :

(1) $h = 6.626 \times 10^{-34}$ Js

(2) $m_e = 9.1 \times 10^{-31}$ kg.

[This question paper contains 4 printed pages.] 27 NOV 2015

Sr. No. of Question Paper : 6227 F-5 Your Roll No................

Unique Paper Code : 2221501

Name of the Paper : Quantum Mechanics and its Applications I

Name of the Course : **Erstwhile FYUP B.Sc. (Hons.) Physics**

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. **All** questions carry equal marks.

1. Attempt any **five** questions of the following:

(a) Discuss the various postulates of Quantum Mechanics.

(b) Using the operator representation of p_x, p_y and p_z

$[x, p_y] = 0,$

$[p_x, p_y] = 0,$

$[y, p_y] = i\hbar$

(c) Write the general solution of the time dependent Schrodinger equation with time-independent potential.

(d) Estimate the ground state energy of a particle in an one dimensional box of L using uncertainly relation.

(e) Obtain the expression of effective mass of an electron in a metal and use the energy spectrum of electron to discuss whether the effective mass of an electron can be negative.

P.T.O.

(f) Write the expression for ground state energy of a quantum harmonic oscillator and discuss its physical significance.

(g) What is the expectation value of K.E of electron in a hydrogen atom if the ground state wave function of electron in the atom is $Q_{100} = \frac{1}{\sqrt{\pi}} \frac{e^{-r/a_o}}{a_o^{3/2}}$

(h) Prove that $\sigma \times \sigma = 2i\,\sigma$ (3×5=15)

2. (a) For the Gaussian wave packet given by

$$(x, 0) = \frac{1}{(\pi\sigma_o{}^2)^{1/4}} e^{-x^2/2\sigma_o{}^2} e^{ip_o x}/\hbar$$

which describes a particle localized within a distance σ_o moving with an average momentum P_o with a momentum spread approximately equal to $\hbar/\sigma_{o}$.

Evaluate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$ and show that $\Delta x \, \Delta p = \frac{\hbar}{2}$. (10)

(b) Using the time dependent Schrodinger equation in one dimension, discuss the concept of stationary states. (5)

3. (a) Discuss Kronig-Penney model and solve the Schrodinger equation for the motion of electrons in a one dimensional periodic potential leading to energy band structure in a solid. (10)

(b) Using the idea of energy bands, briefly explain the difference between a conductor, a semiconductor and an insulator. (5)

4. (a) Solve the Schrodinger equation for a particle having energy E in a square well potential defined by

$V(x) = V_o$ for $x < -a$

$= 0$ for $-a < x < a$

$= V_o$ for $x > a$

where $E < V_o$

(10)

(b) Discuss the significance of the quantum numbers n, l, m_l and m_s. (5)

5. (a) Set up time independent Schrodinger equation for a particle of mass m performing simple harmonic motion of frequency w. Show that the allowed energy must be of the form

$$E = \left(n + \frac{1}{2}\right)\hbar\omega$$ where n is an integer. (10)

(b) Sketch the wave function and corresponding probability density for the simple harmonic oscillator for n = 1 and 2 states. (5)

6. (a) Write the Schrodinger equation for hydrogen atom in spherical polar coordinates. Split the equation into three equations, separately depending on r, θ, φ dependent wave functions. Obtain the solution of the radial equation. (10)

(b) The wave function for hydrogen atom in 1s state is

$$R_{1s}(r) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_o}\right)^{3/2} e^{-r/a_o}$$

where a_o = Bohr radius.

Calculate the expectation value of potential energy of the electron in this state. (5)

6227

7. (a) Describe Stern-Gerlach experiment. Discuss the significance of this experiment. Why is an inhomogeneous magnetic field required? (10)

(b) A beam of silver atom with a velocity of 10^6 cm/s passes through a magnetic field of gradient 50 $W/m^2/cm$ for a distance of 10 cm. What is the separation between the two components of the beam as it comes out of the magnetic field? (5)

Physical constants:

$h = 6.6 \times 10^{-34}$ Js;

$e = 1.6 \times 10^{-19}$ C

$m = 3 \times 10^{-31}$ kg

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5768

F

Your Roll No................

Unique Paper Code : 222502

Name of the Paper : PHHT-516 : Quantum Mechanics

Name of the Course : **B.Sc. (Hons.) Physics**

30 NOV 2015

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. Use of non-programmable scientific calculator is allowed.
5. Symbols have their usual meaning.

1. Attempt any **five** of the following : (5×3=15)

   (i) The size of the nucleus is of the order of 10^{-15} m. Calculate the uncertainty in the momentum of the electron if it were to exist inside the nucleus.

   (ii) Radiation of a certain wavelength is incident on a metal surface having a work function of 9.7 eV. Electrons are emitted from the surface with a maximum kinetic energy of 2.7 eV. What is the wavelength of the incident radiation ?

   (iii) Find the phase and group velocities of the de-Broglie waves of an electron whose speed is 0.9c.

   (iv) Which is more effective in preventing tunnelling, the barrier potential height (U) or the barrier width (L) ? Why ?

P.T.O.

(v) Mention the conditions required for the physical acceptability of a wave function.

(vi) What are the angles between $\vec{L}$ and the z-axis for $l = 1$?

(vii) A free particle of mass m is described by the normalized wave function

$$\psi(x) = A\, e^{i\mu x},\ A \text{ and } \mu \text{ are constants.}$$

Determine the kinetic energy of the particle.

(viii) An electron and proton have same de-Broglie wavelength. Show that kinetic energy of electron is greater than that of proton.

[Consider non-relativistic case]

2. (a) What is Compton effect ? (2)

(b) Derive an expression for the Compton shift. (8)

(c) Show that the ratio of kinetic energy of the recoil electron to the energy of incident photon is

$$\frac{\xi(1-\cos\phi)}{1+\xi(1-\cos\phi)}$$

where $\xi = \dfrac{hv}{m_0c^2}$, v is the frequency of incident photon, m_0 is the rest mass of electron and ϕ is the angle between the incident and scattered photon.

(5)

3. (a) A wave packet is formed by superposition of two harmonic waves:

$$y_1 = A\cos(\omega t - kx)$$
$$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

Obtain the expressions for the phase and group velocities and hence prove that the relation between them is given by

$$v_g = v_p + k\frac{dv_p}{dk}$$

(5,2)

(b) If a free particle of mass m is moving with speed v, say along positive x–axis, then prove that :

(i) $v_g = v$ and $v_p = \frac{v}{2}$ (non-relativistic case)

(ii) $v_g = v$ and $v_p = \frac{c^2}{v}$ (relativistic case)

here, c is the speed of light. (8)

. . .

4. (a) A free particle of mass m moving in one-dimension (say along positive x-axis) with momentum p and energy E can be described in quantum mechanics by the monochromatic plane wave $\psi(x, t) = Ae^{i(px-Et)/\hbar}$, where A is some constant.

   (i) Obtain the time-dependent Schrödinger equation satisfied by this free particle. (7)

   (ii) How can we normalize the above wave function $\psi(x, t)$? (2)

   (b) Suppose $\psi_1(x)$ and $\psi_2(x)$ are the solutions of one-dimensional time-dependent Schrödinger equation then prove that

$$\psi(x) = a_1\psi_1(x) + a_2\psi_2(x)$$

   is also a solution. Here, a_1 and a_2 are complex constants. (6)

5. (a) A particle of mass m and energy E moves in an infinite potential well:

$$V = \begin{bmatrix} 0, & 0 < x < L \\ \infty, & x < 0 \text{ and } x > L \end{bmatrix}$$

   Determine energy eigenfunctions and energy eigenvalues. Hence, obtain momentum eigenfunctions and momentum eigenvalues. (8,4)

   (b) Find the value of N to normalize the wave function :

$$\psi(x) = \frac{N}{\left(x^2 + b^2\right)^2} \quad -\infty \leq x \leq \infty$$ (3)

6. (a) By solving the time-independent Schrödinger equation for the linear harmonic oscillator (mass = m and angular frequency = ω), prove that eigenfunctions are given by

$$\psi_n(x) = N_n H_n(\beta x) \exp\left(\frac{-\beta^2 x^2}{2}\right), \quad -\infty \leq x \leq \infty$$ (12)

P.T.O.

where, $n = 0, 1, 2, \ldots,$ N_n is some constant and $\beta = \sqrt{\frac{m\omega}{\hbar}}$.

[Note : Solution of $\frac{d^2h}{dy^2} - 2y\frac{dh}{dy} + 2n\,h = 0$, is given by

$h(y) = CH_n(y)$, C is some constant]

(b) Find N_n so that $\psi_n(x)$ is normalized.

$$\left[\text{Use : } \int_{-\infty}^{\infty} H_n^2(y)e^{-y^2}dy = 2^n n!\sqrt{\pi}\right]$$ (3)

7. (a) Write down the time-independent Schrödinger equation for hydrogen atom in spherical polar coordinates r, θ, ϕ. Split this equation into three equations, separately depending on r, θ and ϕ dependent wave functions. (9)

(b) Prove that the average value of r for 1s electron in a hydrogen atom is equal to 1.5 a_0, where a_0 is the Bohr radius. Explain why this average value is greater than Bohr radius ?

$$\left[\text{Given : } R_{10}(r) = \frac{2}{a_0\sqrt{a_0}} e^{-r/a_0} \text{ and } \int_0^{\infty} u^3 e^{-2u}\,du = \frac{3}{8}\right]$$ (5,1)

8. A particle of mass m and energy E moving along positive x-axis is incident on a rectangular potential barrier of width L and height V_0 given by :

$$V = \begin{cases} 0, & x < 0 \\ V_0, & 0 < x < L \\ 0, & x > L \end{cases} \quad (\text{here, } V_0 > 0)$$

Obtain the expressions for the reflection and transmission co-efficients of this particle for the case $E < V_0$. Hence show that R + T = 1 and give its significance. (12,2,1)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5066 **F** **Your Roll No................**

Unique Paper Code : 222563

Name of the Paper : Physics – V : Quantum Mechanics and Atomic Physics (PHPT-505)

Name of the Course : **B.Sc. Physical Science / Applied Physical Sciences**

Semester : V

Duration : 3 Hours Maximum Marks : 75

28 NOV 2015

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. Each question carries equal marks.

1. (a) Describe an experimental arrangement for studying the Photoelectric effect. What results do you expect ? Explain. Describe a phenomenon which is opposite to the photoelectric effect. (6,2)

   (b) Light of wavelength 500 nm falls on a metal of work function 1.5 eV, calculate

   (i) the threshold frequency

   (ii) the maximum energy of photoelectrons and

   (iii) the cut off potential (3,2,2)

2. (a) Derive an expression for the Compton wavelength of a photon scattered by a free electron. Use this equation to find the wavelength of a photon which is scattered through an angle of 0°. (8)

P.T.O.

(b) A particle of mass m with energy E where $E << V_0$ moves in a potential $V(x)$ given by :

$$V(x) = 0 \qquad x < 0$$
$$= V_0 \qquad 0 < x < L$$
$$= 0 \qquad x > L$$

find the transmission probability. (7)

3. (a) Explain with an appropriate diagram the Davisson and Germer experiment. Explain the result of this experiment. What new phenomenon did the experiment establish ? (8)

(b) Find the de-Broglie wavelength for an electron accelerated through 1000V. (3)

(c) Applying Heisenberg uncertainty principle, estimate the minimum energy of a particle in a box of length L. (4)

4. (a) The wave function of a particle at a given time is

$$\Psi(x) = A \sin \pi x/L \text{ for } 0 < x < L$$

where A is a constant. Normalize to find A and calculate the probability of finding the particle in the range $L/4 < x < 3L/4$ at that time. (3,4)

(b) How is a wave group formed ? Describe with an example. (2,2)

(c) The phase velocity of ocean wave is $\sqrt{g\lambda/2\pi}$ where g is the acceleration due to gravity and λ, is the wavelength of wave. What is the group velocity of wave packet of these waves ? (4)

5. (a) Give an account of various quantum numbers associated with vector atomic model. Write down the important features of these quantum numbers. (8)

(b) Consider an atomic electron in the $\ell = 3$ state, calculate the magnitude of the total orbital angular momentum and the allowed value of L_z. (3)

(c) Explain space quantization of orbital angular momentum. (4)

6. (a) Describe with appropriate diagram

(i) Normal Zeeman effect and

(ii) Anomalous Zeeman effect (4,4)

(b) Describe the Stern Gerlach Experiment with a suitable diagram. What was its outcome ? (7)

7. (a) Define Bohr magneton. Show that the spectral line due to 3d $\rightarrow$ 2p transition in hydrogen atom splits into three components due to spin-orbit coupling. (5)

(b) Consider two electrons at n=4 level of an atom with orbital quantum numbers $\ell_1 = 1$ and $\ell_2 = 2$. Use LS coupling to find all possible states and draw the corresponding energy diagram. (5,5)

8. (a) State (i) Pauli's exclusion principle and (ii) the selection rules for LS coupling. (2,2)

(b) Discuss symmetric and anti-symmetric wave functions which of these represents an electron ? (4,1)

(c) Calculate Lande's g-factor for the following states (3,3)

(i) $^1S_0$ and (ii) $^3D_{5/2}$.

P.T.O.

Physical Constants

$\hbar = 1.05 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ Kg

$m_p = 1.67 \times 10^{-27}$ Kg

$m_n = 1.675 \times 10^{-27}$ Kg

This question paper contains 4+1 printed pages]

Roll No.

S. No. of Question Paper : **853**

G

Unique Paper Code : **222502**

Name of the Paper : **PHHT-516 : Quantum Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

Symbols have their usual meaning.

5×3=15

1. Attempt any *five* questions :

(*i*) A particle of mass m is confined to a one-dimensional line of lenght L. From arguments based on the wave interpretation of matter, show that the energy of the particle is quantized.

(*ii*) What is stationary in stationary states of time-independent Schrödinger equation ?

(*iii*) Determine the speed of an electron whose de-Broglie wavelength is equal to its Compton wavelength.

(*iv*) Which is more effective in preventing tunnelling, the barrier potential height (U) or the barrier width (L) ? Why ?

(*v*) A particle of rest mass m_0 is moving uniformly in a straight line with relativistic velocity, βc, where c is the velocity of light in vacuum and $0 < \beta < 1$. What is the phase velocity of the de-Broglie wave associated with the particle ?

P.T.O.

(*vi*) What are the possible values of n, l and m_l for $3p$ electron in a hydrogen atom ?

(*vii*) A free particle of mass m is described by the wave function

$$\psi(x) = Ae^{i\mu x}, \text{ A and } \mu \text{ are constants.}$$

Determine the momentum of the particle.

(*viii*) Differentiate between Photoelectric effect and Compton effect.

2. (*a*) Write the conclusion of each of the following experiments : 3

(*i*) Photoelectric experiment

(*ii*) Davisson-Germer experiment

(*iii*) Franck-Hertz experiment.

(*b*) The work function of a metal is 4.14 eV. What is the maximum wavelength of a photon that can eject an electron from the metal ? 3

(*c*) A photon of frequency v is scattered (at an angle ϕ) by an electron initially at rest. Show that the maximum kinetic energy of the recoil electron is given by

$$K_{max} = \frac{2h\,v\,\xi}{1+2\xi}$$

where $\xi = \frac{hv}{m_0c^2}$, m_0 is the rest mass of electron. 9

3. (*a*) Show that the phase velocity of the de-Broglie waves of a particle of rest mass m_0 and de-Broglie wavelength λ is given by 7

$$v_p = c\sqrt{1+\left(\frac{m_0c\lambda}{h}\right)^2}$$

(b) If the above particle has kinetic energy K, then prove that the expressions for the de-Broglie wavelength of this particle are given by :

(i) $\lambda = \frac{h}{\sqrt{2m_0 K}}$ (non-relativistic case) 3

(ii) $\lambda = \frac{ch}{\sqrt{K(K + 2m_0 c^2)}}$ (relativistic case) 5

(a) A free particle of mass m moving in one-dimension (say along positive x-axis) with momentum p and energy E can be described in quantum mechanics by the monochromatic plane wave $\psi(x, t) = Ae^{i(px - Et)/h}$, where A is some constant :

(i) Obtain the time-dependent Schrödinger equation satisfied by this free particle. 7

(ii) How can we normalize the above wave function $\psi(x, t)$? 2

(b) Suppose $\psi_1(x)$ and $\psi_2(x)$ are the solutions of one-dimensional time-dependent Schrödinger equation then prove that

$$\psi(x) = a_1 \psi_1(x) + a_2 \psi_2(x)$$

is also a solution. Here a_1 and a_2 are complex constants. 6

(a) A particle of mass m and energy E moves inside an infinite potential well :

$$V = \begin{cases} 0, 0 < x < L \\ \infty, x < 0 \text{ and } x > L \end{cases}$$

The normalized wave functions of this particle are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, 0 \le x \le L \text{ and } n = 1, 2, 3, \ldots$$

Prove that the expectation values $\langle xp_x \rangle$ and $\langle p_x x \rangle$ in the nth state are related by

$$\langle xp_x \rangle_n - \langle p_x x \rangle_n = i\hbar .$$ 6

P.T.O.

(*b*) Using $\psi_n(x)$, obtain momentum eigenfunctions and momentum eigenvalues of this particle. 5

(*c*) Find the probability of finding this particle in the range $0 \le x \le L/n$ when it is in the n^{th} state. 4

6. One-dimensional harmonic oscillator (mass = m and angular frequency = ω) is in the ground state given by

$$\psi_0(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} \exp\left(\frac{-\beta^2 x^2}{2}\right), -\infty \le x \le \infty \text{ and } \beta = \sqrt{\frac{m\omega}{\hbar}}$$

(*a*) Prove that in this state

$$\Delta x \, \Delta p_x = \frac{\hbar}{2}$$ 10

$$\left[\text{Note}: \Delta y = \sqrt{\langle y^2\rangle_0 - \langle y\rangle_0^2}, \langle f\rangle_0 = \int_{\infty}^{\infty} \psi_0^*(x) f \psi_0(x) dx,\ p_x = -i\hbar\frac{\partial}{\partial x}\right.$$

$$\left. \text{and } \int_0^{\infty} u^2 e^{-u^2} du = \frac{\sqrt{\pi}}{4}\right]$$

(*b*) Determine the probability of finding this harmonic oscillator in the classically forbidden region. 5

7. (*a*) The radial Schrödinger equation for hydrogen atom is given as 12

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}\left[E - V - \frac{\hbar^2 l(l+1)}{2mr^2}\right]u = 0$$

where $u(r) = r\,R(r)$ and $V(r) = \dfrac{-e^2}{4\pi\varepsilon_0 r}$

Prove that the radial wave function, $R_{nl}(r)$ is given by

$$R_{nl}(r) = Ae^{-\gamma r/2}(\gamma r)^l L_{n+l}^{2l+1}(\gamma r),\quad n = 1,2,3,\ldots$$

where, $\gamma = \frac{2}{na_0}$, $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2}$ is Bohr radius and A is some constant.

[Note : Solution of $y\frac{d^2h}{dy^2} + (2l + 2 - y)\frac{dh}{dy} + (n - l - 1)h = 0$, is given by

$h(y) = \text{C}\text{L}_{n+l}^{2l+1}(y)$, C is some constant]

(b) Verify that the most probable value of r for $1s$ electron in a hydrogen atom is equal to a_0. 3

$$\left[\text{Given} : \text{R}_{10}(r) = \frac{2}{a_0\sqrt{a_0}} e^{-r/a_0}\right]$$

(a) A particle of mass m and energy E moves in a finite potential well :

$$\text{V} = \begin{cases} 0, 0 < x < \text{L} \\ \text{V}_0, x < 0 \text{ and } x > \text{L} \end{cases} \qquad (\text{here, } \text{V}_0 > 0)$$

Show that the bound state energies ($\text{E} < \text{V}_0$) are given by equation 7

$$\tan k\text{L} = \frac{2kk'}{k^2 - k'^2}$$

where, $k = \sqrt{\frac{2m\text{E}}{\hbar^2}}$ and $k' = \sqrt{\frac{2m(\text{V}_0 - \text{E})}{\hbar^2}}$

(b) Use the Schrödinger equation to obtain the expressions for the reflection and transmission co-efficients of a particle of mass m and energy E, approaching a potential step of height V_0 for the case $\text{E} < \text{V}_0$. 8

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **7127**

Unique Paper Code : **2221602**

F-6

Name of the Paper : **Quantum Mechanics and Application-II**

Name of the Course : **B.Sc. (H), Physics, Erstwhile FYUP**

Semester : **VI**

Duration : **3 Hours** 20 MAY 2016 Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *five* questions.

All questions carry equal marks.

1. (*a*) Define adjoint of an operator in the *bra* and *ket* notations and explain the properties of Hermitian operators.

(*b*) Show that :

$$[\mathrm{H}, p_x] = \frac{-\hbar}{i}\left(\frac{d\mathrm{V}(x)}{dx}\right),$$

where H is the Hamiltonian.

(*c*) State and prove Ehrenfest's theorem.

(*d*) What are Singlet and Triplet states ? Explain with diagrams.

(*e*) The quantum numbers of electrons in a two-electron system are $l_1 = 2, s_1 = 1/2$ and $l_2 = 3, s_2 = 1/2$. Assuming j-j coupling between them, find all the possible states. 5×3

P.T.O.

2. (a) (i) Is the state $\psi = e^{-3i\phi} \cos\theta$ an eigenfunction of :

$$A_\phi = \partial / \partial\phi \text{ or of } B_\theta = \partial / \partial\theta$$

(ii) Are A_ϕ and B_θ Hermitian ?

(iii) Find the commutator $[A_\phi, B_\theta]$.

(b) Derive the effect of the operations of j_+ and j_- operators on the eigenstates of j^2. 6,9

3. (a) Solve the Harmonic Oscillator problem using ladder operators.

(b) State and prove Schwartz inequality. 12,3

4. (a) Explain the Normal Zeeman splitting of spectral lines due to the $2p$ and $3d$ levels using the energy level diagram.

(b) Derive the expression for the Spin-Orbit interaction energy and show that the doublet separation decreases as the quantum numbers 'n' and 'l' increase and that it is also directly proportional to the fourth power of the periodic number of the atom. 6,9

5. (a) Calculate by the variation method, the ground state energy of a Helium atom explaining your choice of the wave function.

(b) Evaluate the Lande 'g' factor and the total magnetic moment for the $^2P_{3/2}$ and $^2D_{3/2}$ states. 12,3

Write short notes on any *three* of the following : 3×5

(a) Pauli's exclusion principle

(b) Rotational and Vibrational spectrum

(c) L-S coupling

(d) Matrix representation of operators in different choices of basis vectors.

(e) Measurement of an observable and collapse of a state vector.

2017

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5774 **II**

Unique Paper Code : 222502

Name of the Paper : PHHT-516 : Quantum Mechanics

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. Use of non-programmable scientific calculator is allowed.
5. Symbols have their usual meaning.

1. Attempt any **five** of the following : (5×3=15)

(a) What is the minimum uncertainty in the energy state of an atom if an electron remains in this state for 10^{-8} s?

(b) Using the expression $[x, p_x] = i\hbar$, find $[x, L_y]$.

P.T.O.

(c) A proton is moving non-relativistically having kinetic energy 1 MeV. Find its de-Broglie wavelength.

(d) Determine whether the following wave functions are physically acceptable solutions of Schrodinger wave equation or not :

(i) Ae^{x}

(ii) Ae^{-x}

(iii) Ae^{-x^2}, $-\infty \leq x \leq \infty$

(e) Find the probability that a particle in a box L wide can be found between $x = 0$ and $x = L/2$ when it is in the first excited state.

(f) For hydrogen atom what are the possible values of l and m_l for n.= 2?

(g) The azimuthal wave function for the hydrogen atom is

$$\Phi(\phi) = Ae^{i m_l \phi}, \quad 0 \leq \phi \leq 2\pi.$$

Find the normalization constant A.

(h) An electron in H-atom is in the 3p state. Which downward transitions (1s, 2s, 2p) are forbidden by the selection rules?

2. An x-ray photon of wavelength 0.05 nm strikes a free electron at rest and the scattered photon departs at 90° from the initial photon direction.

(a) Determine the momenta of the incident photon, the scattered photon and the scattered electron.

[Given: $\tan^{-1}(0.9542) = 43.66°$, $\sin(43.66°) = 0.69$]

(2,4,6)

(b) Determine the Kinetic energy of the scattered electron.

(3)

3. (a) Suppose an electron at rest absorbs the incident photon and moves with the speed v along the direction of incident photon. Using the laws of momentum and energy conservation, determine the value(s) of v. (6)

(b) The photoelectric threshold wavelength for a material is 5000 Å. Find

(i) the work function of this material

(ii) the maximum kinetic energy of the photoelectrons if light of 4000 Å strikes the surface of this material

(iii) the stopping potential for 4000 Å photons

(3,4,2)

4. (a) Determine the phase velocity and group velocity of the wave corresponding to a de Broglie wavelength of $\lambda, = h/p = h/mv$.

(3,5)

(b) A free particle of mass m is described by a wave function

P.T.O.

$\psi(x) = e^{ipx/\hbar}$; p is the momentum of the particle

prove that the probability current density is equal to the speed of this particle. (7)

5. (a) A particle of mass m moves inside an infinite potential well :

$$V = \begin{cases} 0\,, & 0 < x < L \\ \infty, & x < 0 \text{ and } x > L \end{cases}$$

Find the energy eigenvalues and the normalized wave functions of this particle. (10)

(b) Determine the average of squared-momentum, $<p^2>$ of this particle, when it is in the ground state. (5)

6. (a) A particle of mass m is moving in a harmonic potential well,

$$V(x) = \frac{1}{2} m\,\omega^2\,x^2, \; -\infty \le x \le \infty.$$

If this particle is described by a wave function

$\psi(x) = A\,x\,e^{-m\omega x^2/2\hbar}$, then find

(i) A

(ii) Energy of this particle in the given state. (4,6)

(b) Determine the probability of finding this harmonic oscillator in the classically forbidden region, if it is in the ground state. (5)

$$\left[\textit{Given: } \psi_0(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} \exp\left(\frac{-\beta^2 x^2}{2}\right), \; -\infty \le x \le \infty, \; \beta = \sqrt{\frac{m\omega}{\hbar}}\right.$$

$$\left. \textit{and } \frac{2}{\sqrt{\pi}}\int_0^1 e^{-u^2}\,du = 0.843, \; \frac{2}{\sqrt{\pi}}\int_1^\infty e^{-u^2}\,du = 0.157\right]$$

(a) Prove that z-component of angular momentum operator is given by

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$$ (6)

(b) An electron in hydrogen atom is in the state $\psi(\theta, \phi) = A\sin^2\theta\, e^{i2\phi}$, find

(i) A,

(ii) L, the magnitude of angular momentum

(iii) L_z, the magnitude of z-component of angular momentum

$$\left[\textit{Given: } \hat{L}^2 = -\hbar^2\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \operatorname{cosec}^2\theta\frac{\partial^2}{\partial\phi^2}\right)\right]$$

(3,4,2)

P.T.O.

8. A particle of mass w and energy E moves from a region of potential V_0 towards the region of zero potential, as shown below in Fig. (1).

(a) Explain why energy of this particle should be greater than V_0.

(b) Derive the expressions for the reflection and transmission co-efficients of this particle. (2,13)

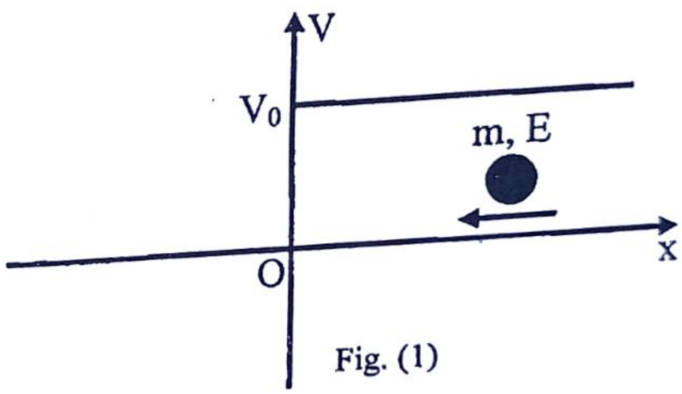

Fig. (1)

Physical Constants:

$h = 6.626 \times 10^{-34}$ J.s $= 4.136 \times 10^{-15}$ eV.s

1 eV $= 1.602 \times 10^{-19}$ J

$m_e = 9.1 \times 10^{-31}$ kg, $m_ec^2 = 0.511$ MeV, $m_pc^2 = 938.3$ MeV

$$\int_0^\infty x^n e^{-ax^m} dx = \frac{1}{ma^{\frac{n+1}{m}}} \Gamma\left(\frac{n+1}{m}\right)$$

(500)

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: 107 I

Unique Paper Code : **32221501**

Name of Paper : **Quantum Mechanics and Applications**

Name of Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.
Q. No. 1 is compulsory.
All questions carry equal marks.

Non-programmable calculators are allowed.

1. Attempt any *five* of the following:

(a) State linearity and superposition principle.

(b) Prove that:

$$[x^n, \hat{p}] = -in\hbar x^{n-1}.$$

(c) What are stationary states? Why are they called so?

(d) What are the conditions for a wavefunction to be physically acceptable?

P. T. O.

(e) What do you mean by space quantization? Explain.

(f) Write the quantum numbers for the state represented by:

$$3^2 D_{3/2}.$$

(g) Define group velocity and phase velocity.

5×3=15

2. (a) Set up the time dependent Schrödinger equation and hence derive the time independent Schrödinger equation.

(b) Derive the expressions for probability density and probability current densities in three dimensions and hence derive the equation of continuity. 7,8

3. (a) Give the theory to explain spreading of a Gaussian wave packet for a free particle in one dimension.

(b) Normalize the following wave function for a particle in one dimension:

$$\begin{cases} A\sin\left(\dfrac{\pi x}{a}\right) & 0<x<a \\ 0 & \text{outside} \end{cases}$$

10,5

4. (a) Solve the Schrödinger equation for a Linear

Harmonic Oscillator to show that the energy eigenvalues are:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega.$$

(b) A Harmonic Oscillator has a wave function which is superposition of its ground state and first excited state normalized eigenfunctions are given by:

$$\Psi(x) = \frac{1}{\sqrt{2}}\,[\psi_0(x) + \psi_1(x)].$$

Find the expectation value of the energy. 10,5

5. Write the Schrödinger equation for a 3D hydrogen atom in spherical polar coordinates. Derive three separate equations for r, θ, φ using the method of separation of variables. Solve the equation for φ to obtain the normalized eigenfunctions and show that they are orthogonal. 15

6. (a) Describe Stern Gerlach experiment with necessary theory. What does it demonstrate?

(b) Explain Normal Zeeman Effect with examples and energy diagram. 8,7

7. (a) What is spin orbit coupling? Calculate the change in the energy levels due to this.

P. T. O.

(b) Show the result of an LS coupling of two non-equivalent p-electrons. 10,5